THE ATOM ©

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INTRODUCTION

The Previous Notes 1, 2, and 3 presented almost all the properties of the electron. This was necessary because it was lack of knowledge of the electron and the related particle structure that allowed certain myths to divert atomic physics towards *statistical ensemble* methods and away from the Main-Line, deterministic picture needed to fill out physics. What is called " classical" physics is capable of describing and *explaining, on the basis of cause and effect*, both atomic and particle physics (and the paradoxes of modern physics) right down to the metaphysical base. There is only one physics, and this and subsequent Notes will attempt to make that clear.

It is not **impossible** to describe what is going on inside atoms. It can be done using only Maxwell's potential equations and Newton's laws. In fact, Quantum Physics does not constitute a physical theory, at the same level as the Theory of Electromagnetism and Newton's laws, any more than Statistical Mechanics did in the 19th century.

This Note will present a planetary type description, of electrons orbiting nuclei, that shows how close Bohr and Sommerfeld came to the correct picture, in spite of the rudimentary awareness of particle properties and structure in the early 1900's. They were unable to see the extended nature of the orbiting electrons, and did not realize that the electron, in turning as it orbited, contributed to the total angular momentum. Thus, they failed to match the orbital angular momentum to the value predicted by Newton's laws. *The standard QM analysis of the atom makes the same error*; and, since Shroedinger's equation *cannot* give the correct answer unless the exactly correct mechanical picture is used to enter the energy of the system, QM carries some of the errors along. Using the extended electron described in the preceding Notes, this will be corrected.

The turning of the orbiting electrons in the ground state ensures that no radiation takes place and that the ground state is stable. Quantization of the orbits is established by the electron's real, doppler shifted longitudinal wave difference frequency. Looking at the atom as a photon generator gives considerable insight into its operation. These and other phenomena are the subject of the present Note.

MYTHS

The Wave Mechanics explanation of the atom's interior has so proliferated that it has deeply ingrained in the modern physicist's mind its hopeless inability to describe that interior. *A firm belief that no deterministic description is possible can be considered the basis of "Modern" physics.* Yet on what grounds? What is the physical evidence? Take the hydrogen atom. An electron, with 98 percent of its energy concentrated inside a sphere of radius 2OO r_e and 99 percent of its charge inside the same sphere, approaches a proton that is essentially the same size as the electron. If the electron is captured, the innermost stable orbit it can occupy is a circle of radius $1.5024 \times 10^5 r_e$. Thus, *the distance between the two particles is greater than 750 times the sphere of significant influence of either one.*

If the electron is *not* captured, it sails past the proton on an hyperbolic orbit, and no one has ever suggested that any serious change in the electron or proton occurs. In the circular and elliptic orbits of capture, the great distance between the particles and the relatively slow motion of the electron argue that just as little change takes place. Then, why the firm belief that the interior cannot be described deterministically? First, a correct description had not been available up until 1989. Second, certain mistaken beliefs related to the application of Maxwell's equations led to the *prediction* of paradoxical non-observed radiation phemomena. Third, a physical reason for the existence of the de Broglie *frequency* has only been forthcoming recently. Beyond that, the fact that the means for measuring the electron's position in the atom without perturbing its motion are not available has been used as a final reason for outlawing a deterministic atom. None of this has any validity.

Considering the proton-electron separation in the atom, no physical reason can be suggested as to why their motion should be any different than outside the atom. In the earlier Notes, the function of the ℓ-waves in establishing the de Broglie frequency was described; and the criterion for when an electron will or will not radiate was presented. Finally, the ability to measure the interior directly is irrelevant to its being deterministic. The perturbation in measurement is a *fact*, not the mysterious concept it has become. Considerable evidence has been collected to show that the particles that make up single atoms retain their simple, individual identities and behave in a straightforward manner.[1] In the next few sections, the structure and operation of the deterministic atom will be described using only ordinary Newtonian mechanics.

THE HYDROGEN ATOM

The simplest atom consists of a single electron orbiting around a single proton. Quantum mechanics gives a proper evaluation of certain aspects of mechanical systems *when the correct description of the mechanics is known*. In the case of the hydrogen atom, past lack of knowledge of the *electron* has caused QM to be used erroneously. The

1. T.Erber et al, "Resonance Fluorescence and Quantum Jumps in Single Atoms", Annals of Physics, 190, p. 254, March (1989). H.Dehmelt, "Experiments on the Structure of an Elementary Particle", Science, 247, pg 539, (1989).

radiated frequencies and total angular momenta predicted are correct; but, because a minute electron angular momentum (due to its extension) has been omitted, the orbital periods and *orbital* angular momenta usually given are incorrect. The progression of the approach from the Bohr-Somerfeld model to the wave mechanics un-picture is so well known that nothing will be said about the history except to point out that the reluctance of textbook authors to give up the visualizable properties of the former for the, until now, more quantitative, non-visualizable latter is significant.

While, in principle, only a complete solution of the total field equations can give an exact picture of this miniature "planetary" system, Newtonian mechanics combined with the properties of the *extended* electron expounded in these Notes gives a highly accurate description of atomic operation. If the "turning" angular momentum is added to the QM analysis, the latter also gives the same results for the *ensemble*.

ORBIT ANALYSIS

Standard planetary analysis begins by considering the motion of a satellite of mass m_0 moving in a central field (see Figure 1), e.g. the Moon

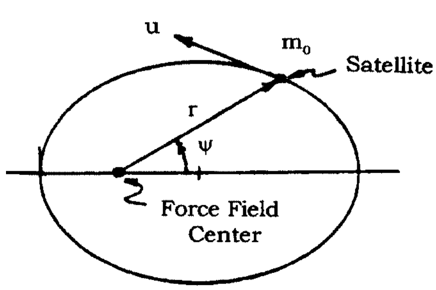

Figure 1. A planar orbit

orbiting a fixed Earth. The presentation here is a modification of Goldstein's treatment of the Kepler problem in astronomy.[1] Customarily, the analysis is carried out with the orbit plane in three dimensions; but for complete *visualization* of details, the orbit plane

1. H.Goldetein, Classical Mechanics, p. 229, Addison-Wesley Press, Inc. Mass. (1950). See also: R.Becker, Electromagnetic Fields and Interactions, Vol 2, p. 56, Dover Publications, N.Y. (1982); G.F.Lothian, Electrons in Atoms, pgs. 65, 74, Butterworths, London (1963); A.Ruark and H.Urey, Atoms, Molecules, and Quanta, Vol 1, Chs 4 and 5, Dover Publications, N.Y. (1964).

will be described with two dimensions. To keep the discussion simple, *even in the atomic case, mass variations will be ignored* ($\gamma = 1$), and will have no important effect on the picture or the principles being presented.

The satellite's energy is defined as $E = T + V$, where $V = -k/4\pi r$ is its potential energy, its kinetic energy is (Heaviside-Lorentz Units),

$$T = \frac{1}{2m_0}\left(p_r^2 + \frac{p^2}{\eta^2 r^2}\right) \quad , \tag{1}$$

p_r is its radial momentum and p its *total* angular momentum. To be defined later, η is unity in the ordinary planetary case. In the simplified Moon-Earth system, the *orbital* angular momentum is,

$$\boxed{p_\psi = m_0 r^2 \dot{\psi}} \quad , \tag{2}$$

and this is usually entered for p in Eq. (1). However, *the Moon always presents the same face to the Earth*, rotating one turn about its own axis for each complete orbit. In the atom, the electron *must* turn that way in all possible orbits (see Note 2), with a *turning* angular momentum (not the spin),

$$p_t = K_t p_\psi \quad . \tag{3}$$

Thus, for a close parallel to the atomic case the planetary example must be visualized with a similar constraint, with the total angular momentum written as,

$$p = p_\psi + p_t = (1 + K_t)p_\psi \quad . \tag{4}$$

Next, Newton's second law is used to write the radial force equation,

$$F_r = m_0\left(\ddot{r} - r(1 + K_t)\dot{\psi}^2\right) \quad , \tag{5}$$

and the angular momentum equation,

$$\frac{dp}{dt} = 0 \quad , \quad p = k_\psi \quad \text{(constant)} \quad . \tag{6}$$

In the general case, for E<0 and attractive force $F_r = -k/4\pi r^2$, a rather long and convoluted derivation[1] leads to closed elliptical orbits ($K_t = 0$) or *almost* elliptical orbits (precessing, K_t <0; recessing, K_t >0), given by,

$$r = \frac{a(1-\varepsilon^2)}{1+\varepsilon\cos(\eta\psi)} \quad , \quad \eta = \sqrt{1+K_t} \quad , \tag{7}$$

with the parameters (Heaviside-Lorentz units),

$E = -\frac{k}{8\pi a}$	Constant energy E for each possible orbit.
$p = k_\psi = \eta\sqrt{\frac{m_0 k}{4\pi}(1-\varepsilon^2)a}$	Constant total angular momentum for each possible orbit.
$a = \frac{r_{min} + r_{max}}{2}$	Similar to the semi-major axis of the elliptical case.
$b = \sqrt{r_{min} r_{max}}$	Corresponds to the semi-minor axis.
$\varepsilon = \sqrt{1-\left(\frac{b}{a}\right)^2}$	Orbit eccentricity parameter.

In terms of these parameters, the radial momentum is given as a function of r by,

$$p_r = \sqrt{2m_0 E + \frac{2m_0 k}{4\pi r} - \frac{k_\psi^{\ 2}}{\eta^2 r^2}} \quad . \tag{8}$$

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1. A.Ruark and H.Urey, Atoms, Molecules and Quanta, Vol. 1, p. 133, Dover Publications, N.Y.(1964). B.Shore and D.Menzel, Principles of Atomic Spectra, p. 45, J.Wiley and sons, N.Y.(1968).

Without the turning constraint, $K_t = 0$ and $\eta = 1$, reducing these equations to the textbook elliptical case. All of this is well known[1], along with the fact that *any choice of semi-major axis a and eccentricity* ε *in the astronomical case (with* $k = Gm_0M$*) will produce a physically realizable orbit* as long as the satellite is far enough away from the force field central mass. Because the energy E is not a function of the eccentricity, any specific choice of a applies to a whole family of pseudo-ellipses and their corresponding total angular momenta p, the largest of which matches the circular orbit with radius a and $\varepsilon = 0$ (see Figure 5).

In the Moon-Earth case, the center to center distance is about 60 times the Earth's radius. Since the electron-proton separation is at least 750 times their significant regions of influence, and these particles exhibit inertia, momentum and all other common properties of ordinary matter, it should be clear that *no obstacle to their executing planetary type orbiting motion exists in any general way*. Therefore, to apply the preceding equations to the hydrogen atom, it is only necessary to set $k = e^2$ (Heaviside-Lorentz units) and supply a rationale for choosing semi-major axis a, ε and K_t. However, it is at this point that the atomic case begins to differ significantly from the astronomical.

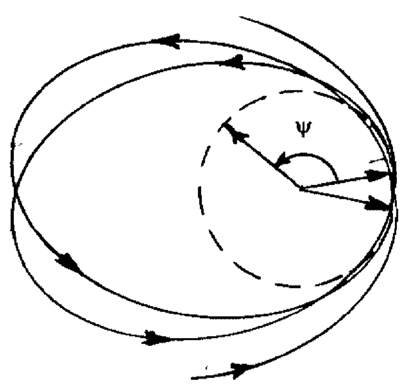

Figure 2. The r_{min} period is less than $\psi = 2\pi$.

For example, in the purely elliptical case, the period of the orbit represents one complete circuit of the ellipse repeated over and over again. In the atomic case, the orbits are not closed but *recess*, as shown in Figure 2. Here, the cycle is considered to go from one r_{min} to the next r_{min}, *which shifts orientation* in the cases where $K_t \neq 0$. Because of

1. H.Goldstein, Classical Mechanics, Addison-Wesley Press, Inc. Mass. (1950).
R.H.Dishington, Physics, Ch.l2, Beak Publications, Pacific Palisades, CA (1989).

this shift, each pseudo-ellipse cycle is completed when ψ has swept out only $2\pi/\eta$ radians.

A great deal of knowledge about atoms comes from their radiation spectra. From the time of Bohr and Sommerfeld, it has been clear that atoms exist in stable or pseudo-stable states; and only when an electron shifts from one orbit to another does radiation occur. Because this was never tied to a cause and effect explanation, but only to the mysterious "quantum", the de Broglie "wave" and "h", the orbits and the visualization were ultimately lost. *Here, the cause and effect chain is traced directly from the properties of the electron and its ℓ-waves,* and the method for finding the semi-major axis values of a that give the observed selected orbits is presented.

THE DE BROGLIE FREQUENCY AND PLANCK'S CONSTANT h

In Note 3, it was demonstrated, using only Newton's laws, that Planck's constant h is a **derived** constant that relates the electron's momentum to the Doppler difference frequency of its front and back longitudinal waves. The following shows the way the difference frequency and the derived constant h enter the orbit analysis.

Refer back to Note 3, on wave-particle duality. There it was shown that when an electron moves along a path at velocity u, its radially outward moving $\ell-$waves are Doppler shifted, resulting in a difference frequency between the front and back waves of,

$$\nu_d = 2\gamma\beta\nu_e \quad , \tag{9}$$

where, $\beta = u/c_0$ and ν_e is the electron's intrinsic $\ell-$wave *rest* frequency (1.2355898×10^{20} cyc/s, as determined by the Rydberg constant).

Eq.(9) and the *linear* momentum p_L were used, in Note 3, to write,

$$p_L c_0 = \gamma m_0 u c_0 = h\frac{\nu_d}{2} \quad , \tag{10}$$

where h is the **derived** Planck's constant (H-L Units),

$$h = \frac{m_0 c_0^2}{\nu_e} = 6.6260755 \times 10^{-27} \quad . \tag{11}$$

However, for the atomic orbit, the derivation of Note 3 must be modified, as follows, to account for the separation of the *orbital* and *turning* angular momenta.

Using Eq.(2) of this Note, the electron's orbital *linear* momentum is,

$$p_L = \frac{p_\psi}{a} = \gamma m_0 u \quad . \tag{12}$$

Combining Eqs.(9) and (12),

$$p_L c_0 = h\frac{\nu_d}{2} \quad , \tag{13}$$

where again h is the **derived** constant of Eq.(11). Eq.(13), when transposed, gives the de Broglie difference frequency[1],

$$\nu_d = 2c_0\frac{p_L}{h} \quad . \tag{14}$$

However, it is the *total* angular momentum,

$$p = p_\psi + p_t = \eta^2 p_\psi \quad , \tag{15}$$

that sets the electron's velocity and the difference frequency. Defining the *total* linear momentum, including the "turning", as,

$$p_{Lt} = \frac{p}{a} = \eta^2 p_L = \eta^2 \gamma m_0 u \quad , \tag{16}$$

Eq.(13) becomes,

$$p_{Lt} c_0 = \eta^2 \frac{h\nu_d}{2} \quad . \tag{17}$$

1. Although the author has chosen to call ν_d the de Broglie frequency, it should not be confused with $\nu_{db} = E/h$, a fictitious frequency of a fictitious wave $\left(\nu_d = 2\frac{u}{c_0}\nu_{db}\right)$.

As discussed in Note 3, *although there is a difference* <u>*frequency*</u>*, there is no difference wave*. For an atomic orbit, the proper inversion of Eq.(17) is,

$$\Lambda_d = \frac{2c_0}{\eta^2 \nu_d} = \frac{h}{p_{Lt}} \quad . \tag{18}$$

where Λ_d is **not** the wave length of a mysterious wave that travels along curved paths. It is determined by real, longitudinal waves that emanate and propagate radially from the electron's center. Λ_d has nothing to do directly with the wavelength of any wave. Instead it is simply *the distance the* <u>*electron*</u> *travels during* $2/\eta^2\beta$ *cycles of the difference frequency*.

THE STEADY-STATE ORBITING ELECTRON FIELD

To better understand the nature of Λ_d in the atom, the total field of an orbiting electron will be visualized. One of the most significant effects of the electron Doppler *difference* frequency (see Note 3) occurs when the electron moves periodically in a closed path in a central electrostatic field. In that situation the conditions for the *total* field solution are quite different from those of the free electron. In the *circular* orbit case, for example, only when the effective circumference of the orbit, $L = 2\pi a/\eta$, is related to the *difference* frequency by,

$$\nu_d = n \frac{2}{\eta^2} \frac{c_0}{L} \quad , \tag{19}$$

where a is the orbit radius, and $n = 1, 2, 3, ...$, can a steady state field solution exist. This can be explained as follows.

An attempt will be made to visualize the overall field surrounding the orbiting electron near the orbit and at great distances from the center. It will be shown that *Eq.(19) is the criterion necessary to maintain the combined solution in steady state*.

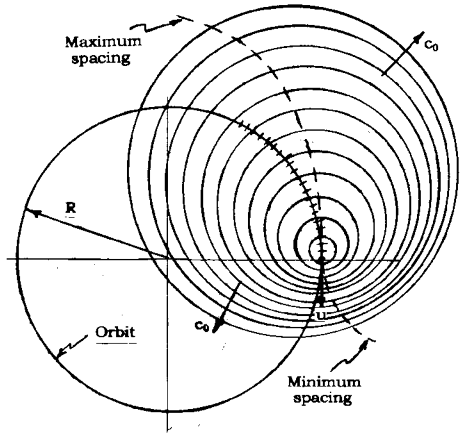

Figure 3
Orbiting electron ℓ-waves

Take the hydrogen ground state orbit as an example. If the electron's ℓ-waves are drawn to scale, because u/c_0 is small, it is impractical to show enough waves to see their centers displaced to match past positions of the particle as it moves along the orbit. A better idea about the minute but significant effects taking place can be obtained by *artificially exaggerating the velocity* u. Then, the effect in space is seen to be a shifting of the positions of maximum and minimum bunching of the $\ell-$waves, as illustrated in Figure 3. Full turning of the electron's field is implicit.

The same exaggerated orbit velocity and, in addition, *artificially reduced wave propagation velocity* allows a plot of the *outer* regions of the field. Figure 4 shows every 2,348th wave front, and the bunching and extending of those fronts can be seen to spiral outward so that the spacing between successive bunching or extending is,

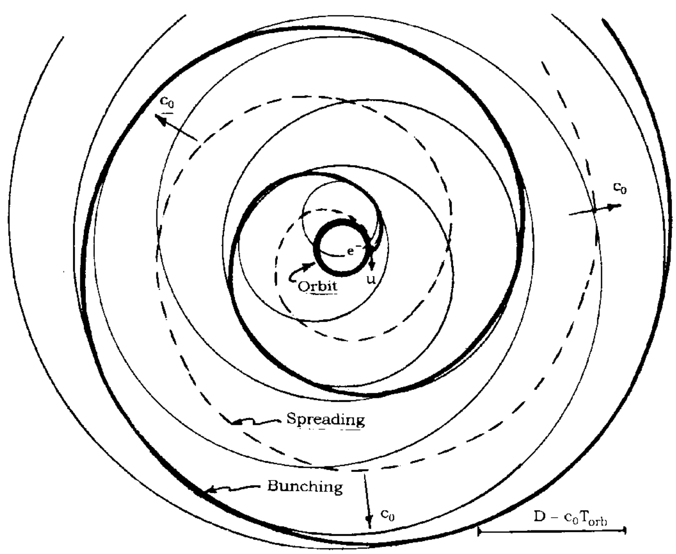

Figure 4
Outer field diverging waves, exaggerated ground state orbit.

$$D = c_0 T_{orb} \quad . \tag{20}$$

By turning the figure in the direction of the electron's motion, the outward motion of the spirals can be seen as a good representation of the total field equation solution for the hydrogen ground state. To be a <u>steady state</u> solution, *the spiral must occupy exactly the same position relative to the orbit, as the electron returns to the same orbit position*, taking into account the orbit recession. This poses no problem in the outer field, as long as the correct phasing occurs along the orbit.

In Figures 3 and 4, the two spirals (bunching and spreading) can be seen to approach the orbit tangentially and to join at the electron center. As the electron orbits, the same picture is repeated at each point. Again this can be seen by simply rotating the plots to simulate the electron's motion. Clearly, depending upon the velocity u and the *effective* orbit circumference L, the phase position of the wave fronts at the electron center may or may not be exactly the same as the electron completes its round trip and returns to the recessed starting point. But, if those phases are not identical, then the outer field will be *changing* and the total field equation solution will ***not*** be <u>steady state</u>.

To find the criterion for a return to the same phase condition, think of a point on the orbit just as the electron is passing. First, a series of front waves, moving in the direction of the electron's motion, cause an oscillation at the point of frequency ν_f. Then a series of back waves, leaving in a direction opposite to the electron's motion, cause an oscillation at the point of frequency ν_b. *The same phenomenon occurs at each point of the orbit circumference*, differing only in the time when the electron passes. Only if the number of front wave cycles $N_f = \eta^2 \nu_f L / 2c_0$, in the distance equal to the *effective* circumference L, and the number of back wave cycles $N_b = \eta^2 \nu_b L / 2c_0$, in that same distance, are both *integers* can the phases and electron oscillation match at each point. Thus *the condition for a steady-state field in this circular orbit* is that the difference between N_f and N_b is also an integer,

$$n = N_f - N_b = \eta^2(\nu_f - \nu_b)\frac{L}{2c_0} = \eta^2\nu_d\frac{L}{2c_0} \quad , \tag{21}$$

which is just Eq.(19) rearranged.

Thus, those *circular* paths specified by Eqs.(19) or (21) are stable, and the elliptical orbits with the same semi-major axis a are pseudo-stable. The profound effect this has on atomic structure was first pointed out by L. de Broglie, through Eq.(17) of Note 3, although he was inspired by a shrewd guess based on symmetry rather than an understanding of the electron's structure. Unfortunately, the wavelength of the de Broglie frequency has been emphasized, and a mysterious wave of a much different frequency than the actual frequencies, ν_f and ν_b, directly involved has been used to describe the "matter waves" of quantum mechanics. Clearly, the mathematical nature of such waves that can travel in a circle along an orbit that is believed not to exist presents a problem to anyone interested in physics. The preceding picture gives a much more realistic description of the phenomena in three dimensions.

THE SEMI-MAJOR AXIS a

It is fortunate that the solution for the orbital energy E is degenerate for all of the non-circular paths that have the same semi-major axis a, because in all those paths the difference frequency and electron velocity vary, and those cases are not truly steady-state. The criterion for finding the allowed a values can be obtained from the *circular* orbit.

Earlier it was shown that Newton's laws were adequate in the astronomical problem. They provided the solution of Eq.(7), which includes the parameter equation for the *circular* orbit case ($\varepsilon = 0$),

$$p = k_\psi = \eta\sqrt{\frac{m_0 k}{4\pi}a} \quad . \tag{22}$$

In this solution, no further restriction is placed on either the constant total angular momentum p or the orbit radius a. The arbitrary choice of either one determines the other through Eq.(19). It is at this point the atomic case deviates most, for the choices of p and a are not arbitrary in the atom, *since the electron itself imposes another condition on* p. Consequently, p is determined and a follows directly from Eq.(22). Thus, p and a are fixed and not arbitrary.

The criterion for finding the allowed total angular momentum and semi-axis a results from the combination of Eqs.(18) and (19),

$$L = \frac{2\pi a}{\eta} = n\Lambda_d \quad . \quad n = 1, 2, 3, \ldots. \qquad (23)$$

Here, n is the familiar principal "quantum number", *obtained using only Maxwell's equations and Newton's laws*. Only those orbits specified by Eq.(23) are stable or pseudo-stable.

Now, to find the alternative equation for p, combine Eqs.(18) and (23) to read,

$$p = ap_{Lt} = \frac{\eta n h}{2\pi} \quad . \qquad (24)$$

Equating Eqs.(22) and (24) yields,

$$a = \frac{n^2 h^2}{\pi m_0 e^2} \quad . \quad n = 1, 2, 3, \ldots \qquad (25)$$

These values of a, used with the energy parameter in the list following Eq.(7), specify all the allowable total angular momenta, orbit energies and sizes. They do not establish the *shapes* of the elliptical orbits. That will be taken up next.

ORBIT SHAPE AND ECCENTRICITY

The selection rule for *circular* orbits can be rewritten, with the help of Eq.(24) above, in the form,

$$p_{Lt} L = nh \quad , \quad (n = 1, 2, 3 \ldots) \qquad (26)$$

where L is the length of path and p_{Lt} is the *total* linear momentum of the electron along the path. If the orbit is *elliptical* the electron's distortion with changing velocity precludes that orbit's stability. Nevertheless, the rate of radiation is not that great, so a kind of pseudo-stability exists. If the radiation is neglected for the moment, it is clear that the steady state field about the orbit is not the simple cyclic spiral discussed above. However, even when the path is elliptical, it is possible to visualize the outer field spiraling outward in non-circular form, always matching the electron's difference frequency as it speeds up and slows down along the path of the orbit. If the match at each point is instantaneously correct, then the proper pseudo-steady state criterion suggested by Eq.(26) is,

$$\int_{r_{min\,1}}^{r_{min\,2}} \mathbf{p}_{Lt} \bullet d\mathbf{s} = nh \quad , \quad (n = 1, 2, 3 \ldots) \tag{27}$$

where $\mathbf{p}_{Lt}$ is the total linear momentum of the electron,

$$\mathbf{p}_{Lt} = m_0 \dot{r} \hat{\mathbf{r}} + m_0 (1 + K_t) r \dot{\psi} \hat{\psi} = p_r \hat{\mathbf{r}} + \frac{p}{r} \hat{\psi} \quad ,$$

and $d\mathbf{s}$ is the differential displacement of the electron along the elliptical path. The integration is carried out over the unclosed section of the ellipse corresponding to the cycle or repetition period from r_{min} to r_{min} and angle of $2\pi / \eta$. In terms of the components, Eq.(27) becomes,

$$\int_{r_{min\,1}}^{r_{min\,2}} \mathbf{p}_{Lt} \bullet d\mathbf{s} = \int p_r dr + \int p d\psi = nh \quad . \tag{28}$$

The components integrated give,

$$\int p d\psi = p \int_0^{2\pi/\eta} d\psi = \frac{2\pi}{\eta} p \quad , \tag{29}$$

and,

$$\int p_r dr = \int_0^{2\pi/\eta} \frac{p \varepsilon^2 \sin^2(\eta\psi)}{\left(1 + \varepsilon \cos(\eta\psi)\right)^2} d\psi \quad . \tag{30}$$

To complete the analysis, it is necessary to ask which orbits are circular and which are elliptical. It can be seen from Eq.(25), that the smallest orbit is specified by $n = 1$, and succeeding values of n give larger and larger paths; but their *shapes* are not indicated by n. In fact, it is easy to visualize a set of orbits for which n would be the same, i.e. the total number of difference frequency beats would be the same around the loop, and yet the shape of the paths could be quite different because of the *changing* de Broglie difference frequency. Some clarification comes from a consideration of Eq.(28), in which, for the same n, the two components of momentum could make different contributions. For example, if the orbit was circular, p_r would be zero and the total momentum contribution would be the angular momentum p. In an orbit of non-circular shape, the radial momentum would not be zero, and the angular momentum ($p = constant$) would be smaller. This would give an elliptical type path with a semi-*minor* axis less than a.

Once the allowed orbit sizes have been found through Eqs.(25) and (28), the shapes of the various orbits, due to differences in the de Broglie frequency, can be found from the <u>components</u> of Eq.(28). It should be emphasized that *it is the whole three dimensional field that must be orbit compatible*. In other words, not only is the total momentum along the orbit path matched to the circumference, but all components of the field must also repeat starting with the new r_{min}. This can only happen if the components obey integer relationships such as,

$$\int p\,d\psi = p\int_0^{2\pi/\eta} d\psi = n_\psi h \quad , \quad \int p_r dr_r = n_r h \quad . \tag{31}$$

Finally, from Eqs.(7), (28) and (31),

$$n = n_r + n_\psi \quad , \tag{32}$$

and,

$$\sqrt{1-\varepsilon^2} = \frac{n_\psi}{n} = \frac{\text{minor axis}}{\text{major axis}} \quad ; \tag{33}$$

where the axes here are loosely equivalent to those of an ellipse. Figure 5 sketches the first few allowed orbits. In order to determine them, it was necessary to specify K_t, which is not a universal constant, but *a different constant for each distinct orbit.* In making the sketches in Figure 5, K_t was assumed to be,

$$K_t = \frac{1}{n_\psi} \quad ; \tag{34}$$

but the reasons for the choice will only be discussed later on. Ultimately,

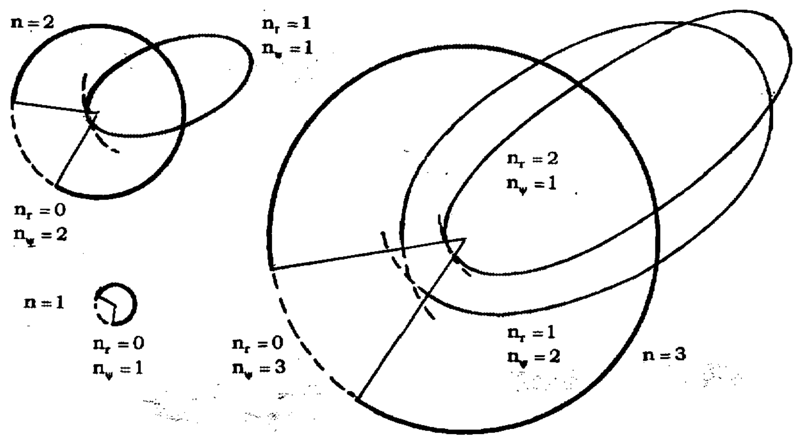

Figure 5. Recessing atomic orbits.

K_t should come as a result of solutions of the total field equations. At present, it is only agreement with experiment that confirms the choice of Eq.(34).

ODDS AND ENDS

Before concluding the specific discussion of hydrogen and broadening the atomic perspective, a few salient points as well as the values and ranges of the various n_i are in order. Just as in the Bohr atom, the principal quantum number can range from 1 to ∞. *Since strong physical reasons have been given to show that circular orbits are basically the most stable,* the lowest value of n_r is zero, or conversely, the maximum value of n_ψ is n. On the other hand, the *orbits corresponding to linear oscillation of the electron through the nucleus, have not been shown* in Figure 5, since it is clear that a large amount of energy is needed to force an electron to approach very close to a proton, and experiments have shown that the proton breaks up and other particles (s quarks) are formed. Also, if the linear orbit were allowed, the electron's rate of radiation would be extreme, and the lifetime of that burst of radiation would be so short as to not qualify this case as even pseudo-stable. In fact, that radiation would present a broad spectrum rather than a line. The upshot of these arguments is that the quantum number ranges are,

$$n = 1, 2, 3 \ldots \infty \quad ; \quad n_\psi = 1, \ldots\ldots, n \quad ; \quad n_r = 0, \ldots\ldots, (n-1) \quad . \qquad (35)$$

Another important item is the omission of the electron's mass variability from the derivation. A proper derivation must include it. It was part of Sommerfeld's model, although too much was left out of that analysis to do more than indicate that the effects were present. Whereas the turning momentum causes the orbit to *recess* a considerable amount, the mass variation has the opposite effect, causing a minute precession. Nevertheless, the effect of the mass variation enters into the energy in a different way and so leads to a measurable effect. Finally, a word should be said in connection with the "Correspondence Principle" used by Bohr, et al. Here *there is no need for it.* In the early days, because the Thomson atom, with its linearly oscillating electrons provided discrete radiation to be identified with the observed spectral lines, the model makers were looking for a match between the mechanical oscillation frequencies and the radiation. In the Bohr model, this only occurred for the outermost orbits, and was interpreted as an asymptotic approach to the "classical" picture. Here, with hindsight, it is known that the lines are radiation emitted during a transition of the electron from one orbit to another. In a typical shift down from one orbit to the next, the electron starts in orbit 1 in a configuration that is stable with respect to de Broglie match and velocity, so that only the persistent buffeting of the zero point fluctuations and its own speed changes perturb the electron's motion. Gradually, the motion reaches a deviation that results in enough electron radiation to prevent recovery of the exact orbit and a slow spiral inwards commences. Soon, the de Broglie match is badly broken and the electron shape oscillates with velocity to produce faster inward spiraling and greater radiation per cycle. Finally, as it approaches closer to the inner orbit, the radiation lessens, the spiraling is slower, although the new velocity is greater, and the electron slowly settles into orbit 2.

It would be surprising if the radiation had the frequency of one of these two orbits. Rather it is logical to suspect that it should be a line of some width, centered perhaps close to the average frequency between the two. In fact, the original Bohr model predicts just such a thing. The presently corrected analysis does so as well. Table I lists the parameters of the first five circular orbits of hydrogen. The radii are essentially those predicted by Bohr, *but the angular velocities of the electron in orbit are*

TABLE I

THE HYDROGEN ATOM

n	R	$\omega_n = \dot{\psi}$	$\omega_{mn} = \frac{\omega_m + \omega_n}{2}$	ω_{mn} (measured)
	(cm)	(sec^{-1})	(sec^{-1})	(sec^{-1})
1	5.2918×10^{-9}	2.9232×10^{16}		
			1.6726×10^{16}	1.5495×10^{16}
2	2.1178×10^{-8}	4.2193×10^{15}		
			2.7727×10^{15}	2.8702×10^{15}
3	4.7651×10^{-8}	1.3260×10^{15}		
			9.5188×10^{14}	1.0046×10^{15}
4	8.4714×10^{-8}	5.7776×10^{14}		
			4.3983×10^{14}	4.6496×10^{14}
5	1.3237×10^{-7}	3.0191×10^{14}		

calculated from the Eqs.(25), (4) and (2). The average ω_{mn} are calculated and the measured values are also given. These latter are the lowest terms of the Lyman, Balmer, Paschen and Brackett series respectively. It is clear that the calculated average ω_{mn} is always less than 8% different from that measured in the line spectrum. Except for the innermost transition, the measured frequency is always higher than the calculated average, which suggests that the electron spends more time or oscillates more vigorously nearer the inner orbit. None of these details pose a problem to the intuition; so the operation of the hydrogen atom is "classical" right down to its innermost orbit (even by the Bohr Theory), making the "correspondence" principle unnecessary.

It is important to realize that *none of the orbiting rates given in Table I match those given by the standard solution of Shroedinger's equation*. For example, the value $2.9232\times10^{16}\,\mathrm{rad/s}$ for the ground state orbit represents the actual, Newtonian orbiting rate found from Eq.(2). In QM, the total angular momentum p is called the *orbital* angular momentum, and the existence of the electron's angular momentum is not recognized although it is unconsciously included. If the proper mechanical format for the energy, separating the orbital and electron angular momenta, is used as the energy in Schroedinger's equation; then, also taking into consideration the present discussion of the linear orbit vs. the circular, the QM result agrees with Table I.

It is hoped that the details of the last few paragraphs will not obscure the fact that the numbers in Table I, for example, are not in any way final. They were obtained from the foregoing equations, ignoring such subtleties as the reduced mass resulting from proton motion. Correct procedure would also consider the turning energy of the proton. Table I overlooks this detail in the calculation of ψ and ω_{mn} (average).

ORBITAL ANGULAR MOMENTUM

The main difference between the preceding development and the Bohr-Sommerfeld atom is in the interpretation of the angular momentum and its effect on the type of orbits, etc. The *orbital* angular momentum is taken here to be p_ψ of Eq.(2) which corresponds to the usual mechanical planetary concept of orbital momentum. In the B-S model, there was no other angular momentum. In the present case, however, there is included the additional angular momentum of the electron's *turning*. According to Eqs.(3) and (34), the added turning angular momentum is,

$$p_t = \frac{1}{n_\psi} p_\psi \qquad (36)$$

The total angular momentum is then,

$$p = p_\psi \left(1 + \frac{1}{n_\psi}\right) \quad . \qquad (37)$$

By uniting Eqs.(29), (31) and (37),

$$p_\psi = \frac{1}{\eta}\frac{n_\psi h}{2\pi} = \frac{n_\psi}{\sqrt{1+\frac{1}{n_\psi}}}\frac{h}{2\pi} \quad ; \qquad (38)$$

so that when Eqs.(37) and (38) are combined, the total angular momentum is seen to be,

$$p = \sqrt{n_\psi(n_\psi + 1)}\ \frac{h}{2\pi} \quad , \qquad (39)$$

a very familiar form. Since its first appearance in the early days of wave-mechanics, it has entered into every situation where angular momentum is present; and has remained disturbing to student and thinking physicist alike, with its forced acceptance, unexplained, take it or leave it. It is now clear that it represents the inevitable form taken when orbital and *turning* angular momenta are combined. Now since electrons, and all other fundamental particles in which distortion energy is represented by a ϕ distribution, have full turning when moving along curved paths in electric fields, the form of the angular momentum will always be that of Eq.(39). It corresponds to the orbital kinetic energy plus the turning energy,

$$T_{\text{total ang. mom.}} = T_\psi + T_t = T_\psi\left(1 + \frac{1}{n_\psi}\right) \quad . \tag{40}$$

It was this condition that led to the choice of $1/n_\psi$ *for* K_t .

THREE DIMENSIONAL ORBIT

In the above solution for the hydrogen atom, only two degrees of freedom were considered; whereas, in real cases, a third degree of freedom is involved. Fortunately, the three dimensional analysis of a hydrogen atom is only slightly more complicated than that given earlier. Since only a few steps are different, the analysis will be sketched here briefly, using the coordinates (r,θ,α) diagrammed in Figure 6.

First, the kinetic and potential energies are,

$$T = \frac{m_0}{2}\left(\dot{r}^2 + \eta^2 r^2\dot{\theta}^2 + \eta^2 r^2 \sin^2\theta\ \dot{\alpha}^2\right) \quad , \quad V = -\frac{e^2}{4\pi r} \quad . \tag{41}$$

The related momenta become,

$$p_\alpha = \eta^2 m_0 r^2 \sin^2\theta\ \dot{\alpha} = k_\alpha \quad ,$$

$$p_\theta = \eta^2 m_0 r^2\dot{\theta} \quad , \quad \text{not constant} \quad , \quad p_r = m_0\dot{r} \quad , \quad \text{not constant} \quad . \tag{42}$$

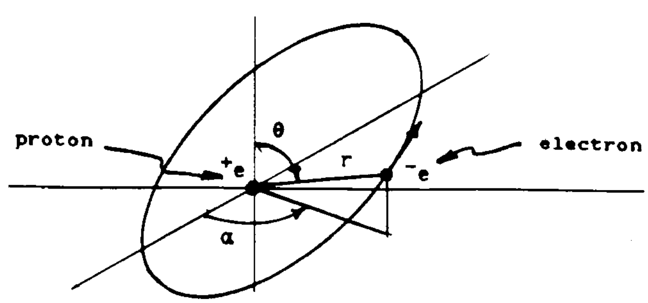

Figure 6. A three dimensional orbit.

Angle α is not in the plane of the orbit, so $p_\alpha = k_\alpha$ represents the z axis component of the *constant* total angular momentum. In the plane of the orbit, the angle is designated by ψ as before, and the total angular momentum p is a constant related to the two components p_α and p_θ by,

$$p = \sqrt{p_\theta^2 + \frac{k_\alpha^2}{\sin^2\theta}} = k_\psi \quad . \tag{43}$$

So all of the results, related to p and a, found earlier for the planar orbit are the same as before. Only the *components* of p present a change from the earlier case.

The p_θ component is the new element in this analysis, and it must satisfy,

$$\int_{\theta_1}^{\theta_2} p_\theta d\theta = \int_{\theta_1}^{\theta_2} \sqrt{k_\psi^2 - \frac{k_\alpha^2}{\sin^2\theta}}\, d\theta \quad , \tag{44}$$

which follows from Eq.(43). Here, again, the angle θ does not go full circle, but rocks between θ_{min} and θ_{max}. A rather convoluted derivation yields,[1]

$$\int_{\theta_1}^{\theta_2} p_\theta d\theta = k_\psi \int_0^{2\pi/\eta} d\psi - k_\alpha \int_0^{2\pi/\eta} d\alpha = (k_\psi - k_\alpha) 2\pi/\eta \quad , \tag{45}$$

since α goes through the same number of radians that ψ does.

Paralleling the earlier planar derivation, the constant total angular momentum p of Eq.(43) shows that *the principal quantum number n acts in the same way*, so the orbit criterion Eq.(27) can be repeated here,

$$\int_{r_{min\,1}}^{r_{min\,2}} \mathbf{p}_{Lt} \bullet d\mathbf{s} = nh \quad , \qquad (n = 1, 2, 3, \ldots) \tag{46}$$

where,

$$\mathbf{p}_{Lt} = m_0 \dot{r}\, \hat{\mathbf{r}} + m_0 \eta^2 r \dot{\theta}\, \hat{\theta} + m_0 \eta^2 r \sin\theta\, \dot{\alpha}\, \hat{\alpha} \quad ,$$

$$\mathbf{p}_{Lt} = p_r \hat{\mathbf{r}} + \frac{p_\theta}{r} \hat{\theta} + \frac{p_\alpha}{r \sin\theta} \hat{\alpha} \quad .$$

1.H.Goldstein, Classical Mechanics, p 301, Addison-Wesley Press, Inc. (1950).

In terms of the components, Eq.(46) becomes,

$$\int_{r_{min\,1}}^{r_{min\,2}} \mathbf{p}_{Lt} \bullet d\mathbf{s} = \int p_r dr + \int p d\psi = \int p_r dr + \int p_\theta d\theta + \int p_\alpha d\alpha = nh \quad . \qquad (47)$$

Again, because the total field equation solution is *cyclic in three dimensions*, each of the components of the motion must return to the cycle start condition to be steady-state; so,

$$\int p_r dr = n_r h \quad , \quad \int p_\theta d\theta = n_\theta h \quad , \quad \int p_\alpha d\alpha = n_\alpha h \quad . \qquad (48)$$

Combining Eqs.(47) and (48),

$$n = n_r + n_\psi = n_r + n_\theta + n_\alpha \quad , \qquad (49)$$

where the quantum number ranges are,

$$\begin{aligned} &n = 1, 2, 3, \ldots, \infty \quad , \quad n_\psi = 1, 2, 3, \ldots, n \quad , \quad n_\theta = 1, 2, 3, \ldots, (2n_\psi - 1) \quad , \\ &n_\alpha = -(n_\psi - 1), -(n_\psi - 2), \ldots, 0, \ldots, +(n_\psi - 1) \quad . \end{aligned} \qquad (50)$$

The limits have been determined purely by physical reasoning which is somewhat more obvious if a more conventional, completely equivalent set of quantum numbers is used. These are,*

Principipal q. n.	$n=1,2,3,\ldots,\infty$	(radius a, etc.)	(51)
Angular Mom. q. n.	$\ell = 1, 2, 3, \ldots, n$	(orbit shape*)	
Magnetic q. n.	$m_\ell = -(\ell - 1), -(\ell - 2), \ldots, 0, \ldots, +(\ell - 1)$	(orbit orientation)	

where,

$$\ell = n_\psi = n_\theta + n_\alpha \quad , \quad m_\ell = n_\alpha \quad , \qquad (52)$$

and the orbit shape is set by,

$$\sqrt{1 - \varepsilon^2} = \frac{n_\theta + n_\alpha}{n} = \frac{\ell}{n} \quad . \qquad (53)$$

It is customary to call the second quantum number (conventionally $\ell - 1$, as ℓ is used here) the orbital quantum number, but more

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* WARNING: ℓ is <u>not</u> the conventional orbital quantum number, but is increased by 1.

realistically ℓ *represents the total angular momentum* of the electron, both orbital and turning (not spin), through the relationship in Eq.(39) repeated here,

$$p = \sqrt{\ell(\ell+1)}\frac{h}{2\pi} \quad . \tag{54}$$

This can be expressed as,

$$p = \ell\sqrt{1+\frac{1}{\ell}}\frac{h}{2\pi} = \eta\frac{\ell h}{2\pi} \quad . \tag{55}$$

The *orbital* angular momentum is,

$$p_\psi = \frac{1}{\eta}\frac{\ell h}{2\pi} \quad . \tag{56}$$

Physically, the circular orbit where $\ell = n$ is certainly a reality, and the $\ell = 0$ case, as discussed earlier, is not regarded as a pseudo-stable state, so the range of ℓ is from 1 to n.

The third, or magnetic quantum number is not significant in any single *free* hydrogen atom because the one electron can take any of the orbits allowed by n and ℓ with any orientation in space; except that, because of the proton's magnetic moment, there is a tendency for the orbit to be perpendicular to the magnetic field lines with the magnetic moment of the electron orbit oriented in the direction opposite the proton's magnetic moment. Physically, this is a very small effect that is normally overshadowed by other conditions. In certain experiments where hydrogen atoms are exposed to an external magnetic or electric field, the external field provides an axis relative to which the various orbits designated by m_ℓ can be shown to be occupied by the hydrogen atom's single electron.

MAGNETICS

So far, nothing has been said of the atom's *magnetostatic* field, although it plays a significant role in both the atom's structure and in its radiation process. The magnetostatic field of a single, circular filament of

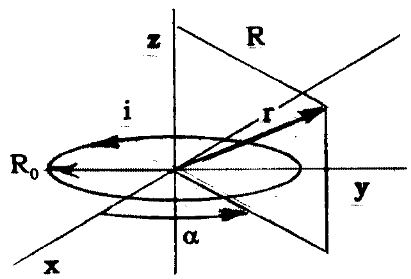


Figure 7. Loop coordinates.

current is often used as a starting point for the description. Figure 7 illustrates some of the coordinates used to describe the field, the structure of which is well documented in the literature. *The form of the equation presented here, however, is simpler than the one commonly used.* It was derived by Heflinger[1]; and is written,

$$\mathbf{A} = \hat{\alpha}\frac{i}{\pi}\sqrt{\frac{R_0}{kR}}\left[K(k) - E(k)\right] \quad , \tag{57}$$

where i is the loop current (hl amp) and K(k) and E(k) are the standard, complete elliptic integrals. However, *the parameter k is different from that used in several widely known references*, and is defined as,

$$k = \zeta\left(1 - \sqrt{1 - 1/\zeta^2}\right) \quad , \tag{58}$$

where,

$$\zeta = \frac{1 + (R/R_0)^2 + (z/R_0)^2}{2R/R_0} \quad . \tag{59}$$

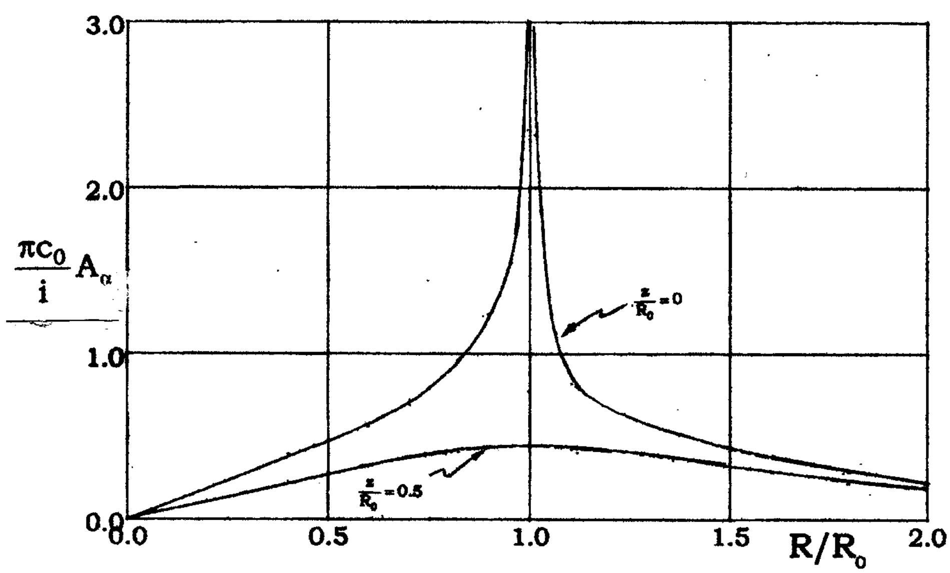
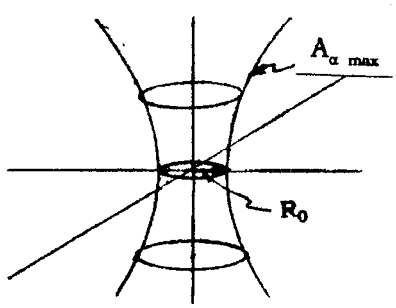


Figure 8. Magnetic field of a single current loop

1. L.O. Heflinger, private communication.

In the plane of the loop ($z = 0$), this field bears some resemblance to the solenoid field described in Note 1. Figure 8 portrays the field distribution for two values of z, zero and $0.5R_0$. Since the current in the filament is actually driving the field, in the $z = 0$ plane the maximum of $\mathbf{A}$ occurs at the radius R_0 of the filament; and the flow tapers off outside R_0. However, the field differs from that of the solenoid because there is no enhancement, by neighboring current loops, to multiply the vortex effect. Therefore, the field is weaker, the rigid body turning in the central region does not extend very far out, and the maximum $\mathbf{A}$ occurs at greater R values for larger values of z. The sheet representing $\mathbf{A}_{max}$ is drawn in Figure 9.



Maximum A_α surface
Figure 9.

Figure 7 shows that, $r = \sqrt{R^2 + z^2} = R / \sin\theta$, so where $r >> R_0$, Eq.(57) can be approximated by,

$$A_\alpha \cong \frac{i}{4\pi} A \frac{R}{r^3} \quad , \qquad (60)$$

where, A is the area πR_0^2 enclosed by the loop of current. This relationship indicates that the field far from the loop has the characteristics of a magnetic dipole of strength,

$$\mu = \frac{i}{c_0} A \quad , \qquad (61)$$

leading to the expression,

$$A_\alpha \cong \frac{c_0}{4\pi} \mu \frac{R}{r^3} \quad . \qquad (62)$$

In vector form,

$$\mathbf{A} \cong \frac{c_0}{4\pi} \frac{\boldsymbol{\mu} \times \mathbf{r}}{r^3} \quad . \qquad (63)$$

THE ATOMIC MAGNETOSTATIC FIELD

In a future Note, the structure of the photon will be investigated, and most of the clues leading to its visualization come from the process of its generation and the machine that produces it. The latter, of course, is the atom; and one most influential component of atoms, related to photon generation, has not yet been described. It is a magnetic field. Generally, not much is said about magnetic effects in atoms. However, this particular magnetic field is well known to produce several significant effects.

Consider the hydrogen atom *ground* state, for example. Its single orbiting electron is traveling in a closed loop, and *conventional analysis assumes that it behaves as a single loop of current* of magnitude,

$$i = \frac{e}{T} = \frac{e\omega_n}{2\pi} \quad . \tag{64}$$

The value of ω_n from Table I gives a magnitude for the equivalent current of $i = 7.9216 \times 10^6$ hl amp. At this point one might be tempted to use Eqs.(57) through (61) to determine the magnetic field and moment; but, for several reasons, this leads to a considerable error. Clearly, the actual mechanism of generating the magnetic field is sensitive to the *difference* between the single orbiting electron and a current carrying loop. However, it is clear that the current 7.9216×10^6 hl amp is the correct amount of charge being carried around the orbit per second, so that it is the true current. It is the Eqs.(57) through (61) that do *not* give the true atomic field generated.

To obtain the correct form of the magnetic vortex, the charge distribution of the extended electron of Note 1 must be used to describe the orbiting charge source. The integration of this complicated field has not yet been accomplished. Nevertheless, the effect of the extended turning charge density appears to allow an *effective* current to be defined that can be used in Eqs.(57) through (61) to give the correct field. That effective current is,

$$i_{eff} = \eta i \quad , \tag{65}$$

where η is the factor defined in Eq.(7). For the hydrogen ground state, $\eta = \sqrt{2}$, so the effective current is $i_{eff} = 1.1203 \times 10^{7}$ hl amp . When this value is used in Eq.(61), for example, the magnetic moment of the hydrogen ground state is found to be,

$$\mu_B = 3.287553 \times 10^{-20} \text{ ergs / hlG} \quad , \tag{66}$$

commonly known as the Bohr magneton. Eqs.(61) and (65) together give the correct moments for all the hydrogen atom orbits when the correct η is applied.

THE *FREE* HYDROGEN ATOM

This last section will present the structure of the three innermost sets of orbits and their total momenta (including spin) and magnetic moments for a hydrogen atom **free** of any *external* electric or magnetic fields. This is something *that no present textbook can do* because of the non visualizability of the QM ensemble approach. Whenever modern textbooks try to describe the various orbital states, they are forced to immerse the atom in some form of external field. The ensuing complication completely obscures the simplicity of a **free** atom. Here the task is to *visualize a single, deterministic atom in a field free region*.

As discussed above, n is the usual quantum number that fixes orbit *size* a and energy E,

$$a = \frac{n^2 h^2}{\pi m_0 e^2} \quad (A) \quad , \qquad E = -\frac{e^2}{8\pi a} \quad (B) \quad . \tag{67}$$

Orbit *shape* is set by n_ψ. The electron is spread out *laterally*, because of its kinetic energy, and has an oblate spheroid shape. Also, in the electric field of the proton, it "turns" (in addition to the spin) so its shape axis is along the orbit. The *spin* aligns itself with the electron *shape* axis.

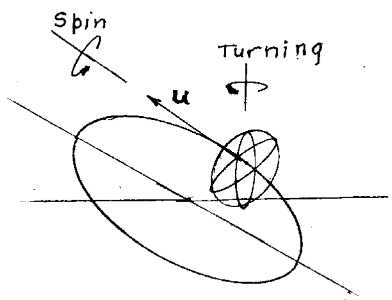


Figure 10.

Figure 10 shows the electron, moving in orbit at velocity **u**, turning and spinning.

TOTAL VECTOR ANGULAR MOMENTUM

The basic *orbital* angular momentum is,

$$p_\psi = m_0 a^2 \omega \quad , \qquad (68)$$

as predicted by Newton's laws. However, because the electron's shape "turns", in the plane of the orbit, one turn per electron period, this "turning" angular momentum adds to p_ψ to give a total angular momentum,

$$p = p_\psi + p_t = \left(1 + K_t\right) p_\psi \quad . \qquad (69)$$

When atoms are *quiescent*, the turning factor K_t is related to the orbit shape through,

$$K_t = \frac{1}{n_\psi} \quad . \qquad (70)$$

Combining Eqs.(68), (69) and (70), the total angular momentum for *quiescent* atoms, *before adding the spin* is (see above),

$$p = \sqrt{n_\psi (n_\psi + 1)} \frac{h}{2\pi} \quad . \qquad (71)$$

At this point, before discussing the total vector angular momentum, **J**, and in order to be close to the conventional notation, the substitutions $S = \sigma = \frac{1}{2}\hbar$ $(\hbar = h / 2\pi)$ and $L = p$ can be made. It is customary, in *ensemble* quantum physics, to use the same *form* that appears in Eq.(71) for S and J; i.e. $\sqrt{s(s+1)}\hbar$ and $\sqrt{j(j+1)}\hbar$. However, from the present point of view, *there is no physical justification for this*, because S and J do not have any extra turning components. Therefor, the vector spin and total vector angular momentum become,

$$\mathbf{S} = \hat{\mathbf{S}}S = \hat{\mathbf{S}}\tfrac{1}{2}\hbar \quad , \quad \mathbf{J} = \hat{\mathbf{J}}J \quad ; \tag{72}$$

and there are no magnetically induced precessions of these vectors involved in the *free* atom.

It is clear from Figure 10 that *the spin vector and the orbit vector are always perpendicular to each other*; so, from Eqs.(71) and (72), the total vector angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$ has a magnitude,

$$J = \sqrt{L^2 + S^2} = \sqrt{n_\psi(n_\psi + 1) + \tfrac{1}{4}}\ \hbar \quad . \tag{73}$$

This very simple equation for J will not be found in the literature, since it depends on the free atom analysis not available to modern physics.

Eqs.(67) and (71) give the energy and angular momentum for each orbit. Referring again to Figure 5, Table I lists some of the values related to those orbits. The radii are given in terms of the *Bohr radius*,

$$a_0 = 5.2918 \times 10^{-9} \quad , \quad \text{cm} \tag{74}$$

a convenient unit. The atomic vortex is observed as the atom's magnetic moment,

$$\mu = n_\psi \mu_B \quad , \tag{75}$$

where μ_B is the Bohr magneton. Values of μ for the inner orbits of hydrogen are listed in Table II.

TABLE II

n	n_ψ	a	ω	p_ψ	L = p	J	μ
		cm	s^{-1}	g-cm/s	g-cm/s	g-cm/s	$\frac{\text{ergs}}{\text{hlG}}$
1	1	a_0	2.9233×10^{16}	$0.7071\hbar$	$1.4142\hbar$	$\frac{3}{2}\hbar$	μ_B
2	1	$4a_0$	varies	$0.7071\hbar$	$1.4142\hbar$	$\frac{3}{2}\hbar$	μ_B
	2	"	4.2194×10^{15}	$1.6330\hbar$	$2.4495\hbar$	$\frac{5}{2}\hbar$	$2\mu_B$
3	1	$9a_0$	varies	$0.7071\hbar$	$1.4142\hbar$	$\frac{3}{2}\hbar$	μ_B
	2	"	varies	$1.6330\hbar$	$2.4495\hbar$	$\frac{5}{2}\hbar$	$2\mu_B$
	3	"	1.3260×10^{15}	$2.5981\hbar$	$3.4641\hbar$	$\frac{7}{2}\hbar$	$3\mu_B$

By now it should be clear that statements indicating that Newton's laws and Maxwell's equations do not apply inside the atom are simply not true. Newton's laws and classical mechanics apply without any modification. Maxwell's equations, when recognized as the weak field equations they are, also apply where they should reasonably be expected to. The need to augment Maxwell's equations for application to the strong field regions inside the particles is a normal classical condition that leads to the visualizable planetary form of atom given here.